Global Closure and Decomposition Invariance

A Reconstruction Argument for L^2 Residual Geometry

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Abstract
We argue that, under specified reconstruction assumptions — decomposition-independent residual partitioning, orthogonal/direct-sum reconstruction sectors, Pythagorean residual composition, and invariant residual magnitude under admissible decomposition changes — the resulting residual norm is inner-product / L^2-type by the Jordan–von Neumann route. Under these conditions, quadratic residual geometry need not be independently postulated, but instead emerges as the stable reconstruction geometry associated with decomposition-invariant admissibility reconstruction.

We further argue that admissibility reconstruction is not uniformly conditioned across the admissibility manifold. Near regions where admissible relational support becomes geometrically constrained, local reconstruction charts may develop degeneracy structure, rank deficiency, or unstable inversion behavior. Quadratic residual geometry therefore appears not merely as convenient mathematics, but as the natural conditioning geometry required for stable local reconstruction of globally admissible structure. In weakly conditioned reconstruction sectors, the framework suggests suppression of distinguishability and emergence of effective classicality under admissibility-preserving projection. Under this framework:

- L^2-type geometry is representational rather than ontic,
- orthogonal projection geometry emerges from admissibility reconstruction,
- and representation-neutrality permits invariant admissibility-conditioning structure without privileging any particular reconstruction chart.

This provides a principled route from global closure structure to quadratic residual geometry while preserving adynamical ontology and avoiding ontological commitment to dynamical optimization, computational evolution, or solver realism.

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1. Introduction

Quadratic structures appear throughout modern physics in remarkably diverse settings, including Hilbert-space norms in quantum mechanics, least-action principles, quadratic field energies, orthogonal mode decompositions, Fourier and Parseval identities, spectral geometry, least-squares reconstruction, and variational projection methods. Their recurrence is typically treated either as a basic mathematical fact or as a highly successful formal feature of physical theory without deeper ontological significance.

In this work we explore a different possibility: that quadratic residual geometry arises naturally from the representational requirements imposed by globally admissible relational structure. The proposal developed here begins from a closure-first ontology in which globally admissible relational organization is primary, while geometric reconstruction structure emerges at the level of embedded representation.

Under this interpretation, L^2-type geometry is not taken as ontically fundamental. Instead, quadratic norm structure appears as the natural geometry associated with stable reconstruction of globally admissible configurations by embedded observers who access the admissible manifold only through local or projected descriptions. The resulting geometry reflects the constraints imposed by decomposition invariance, reconstruction stability, and admissibility preservation across representations.

The goal of the present work is therefore not to describe a dynamical optimization procedure performed by nature, nor to elevate least-squares reconstruction into ontology. Rather, the aim is to explain why quadratic geometry repeatedly emerges whenever admissible relational structure must be represented, decomposed, and reconstructed in a decomposition-independent manner.

This paper occupies a specific position within the cRBW/RSE reconstruction stack. It does not replace Projection Residual Geometry (PRG) [1], which establishes the global projection structure of admissibility, nor Closure-Restoration Geometry (CRG) [2], which develops the local differential refinement associated with admissibility restoration. Its role is narrower and prior to both: to explain why the residual metric shared by PRG and CRG naturally acquires quadratic (L^2-type) structure under decomposition-invariant reconstruction assumptions. The present work therefore functions as a reconstruction-theoretic bridge between global admissibility projection and local restoration geometry.

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2. Ontological Starting Point

We begin from four elements of the closure-first framework, noting that they occupy different levels of the ontological and representational stack. Adynamical Global Constraint (AGC) is treated as primitive; NPRF expresses decomposition invariance of admissibility; BBP ($\partial^2 = 0$) and projection-level reconstruction emerge as derived representational consequences within embedded reconstruction schemes.

The present argument is also closely related to Local Projection Sufficiency (LPS) and No Ontic Excess (NOE): the goal is to identify the minimal reconstruction geometry sufficient for stable decomposition-invariant admissibility reconstruction without introducing surplus representational structure.

2.1 Global Closure

Relational structure satisfies a global closure condition: $\partial^2 = 0$ expressing Boundary-of-a-Boundary closure (BBP), developed within the broader closure-first framework of cRBW/RSE. [3] Under this condition, admissible relational configurations possess globally self-consistent closure structure. Closure is understood as an all-at-once admissibility condition on relational organization rather than as the outcome of a dynamical evolution. The admissible configuration is globally constrained from the outset, and closure defines the admissibility structure itself.

2.2 No Preferred Reference Frame (NPRF)

No Preferred Reference Frame (NPRF) states that no decomposition, partition, foliation, basis, coordinate system, or observer-relative reconstruction carries ontic privilege. The present use of NPRF extends earlier relational-blockworld formulations into a generalized decomposition-invariant admissibility framework. [4] [5] [6] Admissibility must therefore remain invariant under admissible representational decompositions. The resulting structure is decomposition-independent and representation-neutral in the sense that admissibility cannot depend on the particular descriptive chart used to encode it.

This invariance requirement extends beyond coordinate covariance alone. It applies generally to the partitioning and reconstruction of relational structure, requiring that admissibility-preserving descriptions remain mutually compatible across equivalent representations.

2.3 Projection-Level Reconstruction

Embedded observers do not access the globally admissible manifold directly. Instead, they construct local or projected descriptions of admissible relational structure. Because such reconstructions are necessarily partial, representational descriptions generically exhibit residual mismatch, incomplete reconstruction, local conditioning limitations, and projection-dependent deviations from exact admissibility.

Representational geometry therefore emerges from the structure of admissibility reconstruction itself. Geometric relations arise not from direct access to ontology, but from the constraints governing stable local reconstruction of globally admissible relational organization.

2.4 Residual Geometry

In a local affine reconstruction chart — or in an embedding where configuration subtraction is defined — we write the admissibility residual vector as

$$r(C) = C - P_{\mathcal{M}}(C). \tag{2.1}$$

Globally, the residual is defined representation-neutrally as the distance to the admissible set,

$$R(C) = \inf_{X \in \mathcal{M}} d(C, X), \tag{2.2}$$

with the vector form understood as local or embedded-chart notation used where the representation admits it. Globally, the quadratic residual functional is defined as

$$J(C) = R(C)^2, \tag{2.3}$$

while in a local affine chart or embedding where $r(C)$ is defined this takes the inner-product form

$$J(C) = \langle r(C), r(C) \rangle. \tag{2.4}$$

This aligns with the infimum-based residual definition in PRG [1] and Principles of Ontic Minimalism [3], under which $P_{\mathcal{M}}(C)$ denotes any minimizing element where the minimizer is non-unique.

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3. Decomposition Independence and Residual Composition

Assumptions of the L^2Reconstruction Argument

1. Residuals admit locally linearized reconstruction sectors near $\mathcal{M}$.
2. Residual sectors decompose into independent direct-sum components.
3. Admissible decomposition changes preserve residual magnitude.
4. Independent sectors combine via Pythagorean additivity.
5. Completion is assumed where Hilbert structure is invoked.

Consider a residual vector structure that admits decomposition into independent admissibility sectors,

$$r = r_1 \oplus r_2 \oplus \cdots \tag{3.1}$$

If admissibility is to remain decomposition-independent under NPRF, then no particular partition or basis decomposition can possess privileged status. The magnitude associated with admissibility deviation must therefore remain invariant under admissible transformations between equivalent representations.

This requirement strongly constrains the allowable composition law for residual structure. Independent residual sectors must combine in a manner compatible with basis-independent magnitude preservation across admissible decompositions. Under the stated assumptions — specifically Pythagorean additivity across independent direct-sum residual sectors (assumption 4) — the resulting norm satisfies the parallelogram law, and the Jordan–von Neumann theorem then identifies it as inner-product induced. Decomposition invariance motivates the stability requirement; Pythagorean additivity is the load-bearing assumption. Orthogonality and rotational invariance arise with the resulting inner-product structure rather than functioning as premises used to derive it.

$$\| r \|^2 = \| r_1 \|^2 + \| r_2 \|^2 + \cdots \tag{3.2}$$

The load-bearing assumption here is Pythagorean additivity across independent residual sectors (assumption 4). Decomposition invariance motivates this assumption by requiring residual magnitude to remain stable across admissible sector decompositions, but does not by itself derive the parallelogram law. Once Pythagorean additivity is imposed, the

resulting norm satisfies the parallelogram law, and the Jordan–von Neumann theorem supplies the inner-product result.

Given decomposition-independent residual partitioning together with Pythagorean additivity across orthogonal reconstruction sectors, the resulting norm satisfies the parallelogram law. By the Jordan–von Neumann theorem, the norm is therefore induced by an inner-product structure. Under these assumptions, quadratic residual composition becomes the natural reconstruction geometry compatible with invariant residual partitioning across equivalent admissible decompositions.

This is precisely the Pythagorean relation characteristic of inner-product geometry. Quadratic norm structure therefore arises from the requirement that admissibility reconstruction remain decomposition-independent across equivalent representations. The privileged role of quadratic composition is closely related to the Jordan–von Neumann characterization of inner-product spaces, in which the parallelogram law determines inner-product-compatible norm structure. [7]

Under this interpretation, the quadratic structure is not imposed as an ontic primitive. Rather, it arises from the geometric requirements of stable admissibility reconstruction. Once decomposition-independent residual composition is required, inner-product structure follows as the natural reconstruction geometry compatible with invariant admissibility partitioning.

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4. Emergence of Inner-Product Geometry

Once quadratic residual composition is established, an inner-product structure follows naturally. Over real reconstruction spaces, the inner product is recovered through the polarization identity

$$\langle x, y\rangle = \frac{1}{2}(\| x + y \|^2 - \| x \|^2 - \| y \|^2). \tag{4.1}$$

with sign convention depending on whether the inner product is taken linear in the first or second argument. For complex reconstruction spaces relevant to quantum-mechanical Hilbert structure, the corresponding complex polarization identity uses the four-term complex form, with signs adjusted according to whether the inner product is taken linear in the first or second argument; for example, under one common convention:

$$\langle x, y \rangle = \frac{1}{4}(\| x + y \|^2 - \| x - y \|^2 + i \| x + iy \|^2 - i \| x - iy \|^2). \quad (4.2)$$

The resulting geometry carries with it the familiar structures associated with inner-product spaces, including orthogonality, projection geometry, stable decomposition, and spectral structure. Hilbert-like reconstruction spaces therefore arise from the reconstruction properties of decomposition-independent admissibility geometry. Strictly, the reconstruction structure derived here first yields a pre-Hilbert inner-product geometry. Hilbert structure follows upon completion where spectral decomposition and functional analysis are invoked. The resulting reconstruction geometry inherits the familiar properties of Hilbert-space analysis and spectral decomposition. [8]

Under this framework, such structures are not introduced as independent ontological primitives. They emerge from the requirement that admissibility reconstruction remain stable and invariant across equivalent decompositions. Orthogonality, least-squares reconstruction, spectral projection, and related quadratic reconstruction procedures consequently appear throughout physics because they provide the natural geometric language for admissibility-preserving reconstruction.

The widespread appearance of Hilbert-like structure in physical theory may therefore reflect a deep representational consequence of admissibility invariance rather than an ontologically fundamental Hilbert geometry.

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5. Orthogonal Projection as Admissibility Reconstruction

Given an admissible manifold $\mathcal{M}$, embedded reconstruction seeks an admissible approximation $P_{\mathcal{M}}(C)$ to a representational configuration C. The associated residual,

$$C - P_{\mathcal{M}}(C), \tag{5.1}$$

must preserve decomposition-independent reconstruction under changes of basis or partition.

When $\mathcal{M}$ is closed and convex (or a closed linear subspace), the residual-minimizing reconstruction is the unique orthogonal projection. More generally, for nonlinear admissible manifolds projection may be local or multi-valued, and the residual is defined through the infimum distance to $\mathcal{M}$. Orthogonality, least-residual reconstruction, and quadratic norm geometry therefore emerge together as mutually linked features of stable admissibility reconstruction. Local tangent-space refinements belong to the CRG layer discussed below. Orthogonal projection arises as the stable minimum-residual reconstruction structure in inner-product approximation theory. [9]

This provides a natural explanation for the recurring appearance of orthogonal projection structures throughout physics. Quantum reconstruction, variational methods, spectral geometry, wave theory, and constrained field systems all repeatedly employ orthogonal decomposition because decomposition-independent admissibility reconstruction naturally selects inner-product projection geometry as the stable representational structure.

Under this interpretation, orthogonal projection is not elevated to ontology. Rather, it appears as the natural reconstruction geometry associated with embedded representations attempting stable approximation of globally admissible relational structure. This reconstruction interpretation aligns with Projection Residual Geometry (PRG) [1], where admissibility deviation acquires a geometric interpretation through projection-induced residual structure.

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6. Admissibility Conditioning and Reconstruction Stability

Let F denote the local reconstruction map with Jacobian DF. If the singular values of DF are $\sigma_1 \geq \cdots \geq \sigma_n$, define the local conditioning number by

$$\kappa_{\text{cond}} = \frac{\sigma_{\max}}{\sigma_{\min}}. \tag{6.1}$$

Reconstruction becomes poorly conditioned as $\sigma_{\min} \to 0$, where small representational perturbations may induce disproportionately large reconstruction deviations.

Admissibility reconstruction need not remain uniformly well-conditioned across the admissibility manifold. In regions where admissible relational support becomes geometrically constrained, local reconstruction charts may develop near-degenerate directions, rank deficiency, unstable inversion behavior, or singular conditioning structure. Similar conditioning phenomena are well known in inverse-problem theory and ill-posed reconstruction geometry. [10] Such behavior reflects limitations in stable local reconstruction rather than any breakdown of the underlying admissible structure itself.

In these regions, small representational perturbations may correspond to disproportionately large reconstruction deviations, and tangent-space conditioning may become increasingly ill-behaved. The resulting instability is geometric in character, arising from the local structure of admissibility reconstruction near conditioning boundaries.

Quadratic residual geometry becomes especially important under such conditions because stable decomposition requires orthogonal partitioning, while orthogonal partitioning in turn depends upon inner-product structure. If admissibility reconstruction is to remain decomposition-independent even in poorly conditioned regions, the reconstruction geometry must preserve stable residual composition across equivalent representations.

From this perspective, L^2-type geometry appears as the natural conditioning geometry associated with stable admissibility reconstruction. The role of quadratic structure is therefore not merely computational convenience, but the preservation of decomposition-independent reconstruction stability in the presence of constrained admissibility support. Such conditioning behavior may be associated with finite admissibility-support structure and global normalization constraints within closure-first relational frameworks. [11]

This interpretation also provides a useful way to understand projection-level suppression phenomena that appear in weakly accessible reconstruction sectors. In some physical systems, substantial underlying structural variation may project into observationally

compressed effective sectors exhibiting Gaussianization, residual attenuation, or effective classicality. [12] Such behavior need not imply absence of underlying structure, but rather limitations imposed by admissibility-preserving reconstruction geometry when accessible projection sectors become weakly conditioned.

In settings involving repeated weak projection, mode averaging, linear response, or noise, L^2-compatible reconstruction may support effective Gaussian descriptions. This should be read as a suggested consequence of additional reconstruction assumptions rather than as a result derived from quadratic residual geometry alone.

Similar projection-level suppression phenomena appear in recent analyses of axion dark matter detection, where effective-mode reconstruction and weak coupling lead intrinsically quantum underlying states to project into observationally compressed effective sectors exhibiting Gaussianization and effective classicality. [13]

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7. Global Capacity and Admissibility Support

Within a closure-first framework, global admissibility is not unconstrained. Admissible relational structure possesses finite support conditions associated with global consistency, normalization, and reconstruction stability. In this context, global capacity refers to admissible relational support structure rather than to computational resources, energetic expenditure, or dynamical storage.

This usage aligns with the Global Capacity Invariants framework [14], where admissible normalization and finite support structure constrain reconstruction without implying computational storage or dynamical resource expenditure.

As configurations approach admissibility-support boundaries, reconstruction behavior may become increasingly unstable. Tangent spaces may develop near-degenerate directions, local reconstruction charts may become rank-deficient, and inversion structure may become poorly conditioned. Such behavior closely parallels instability structures encountered in ill-conditioned numerical reconstruction and near-singular inversion geometry. [15] Such behavior reflects the geometric limits of stable admissibility reconstruction near the boundaries of admissible support.

Under this interpretation, conditioning singularities are best understood as projection-level signatures of approaching admissibility-support limits rather than as ontological discontinuities. The underlying admissible structure remains globally defined, while local reconstruction geometry becomes progressively less stable as admissibility constraints tighten.

This perspective reframes degeneracy phenomena geometrically. Reconstruction instability does not indicate a breakdown of the underlying ontology, but rather the limits of stable embedded representation near regions where admissibility support becomes highly constrained.

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8. Why L^2 Rather Than Alternative Norms?

Alternative norm structures certainly exist, including L^1, L^∞, and generalized L^p geometries. However, these do not generically preserve the full collection of properties required for stable decomposition-independent admissibility reconstruction. In particular, they do not naturally support canonical inner-product structure, orthogonal decomposition, stable spectral projection, and invariant residual partitioning simultaneously.

The special role of L^2 geometry follows from the fact that quadratic norms induce a canonical inner-product structure. Once inner-product geometry is present, orthogonal projection and stable decomposition follow, allowing residual structure to partition consistently across equivalent representations. This becomes especially important near poorly conditioned reconstruction regions, where admissibility reconstruction must remain stable despite local geometric constraints.

From this perspective, orthogonal decomposition is not an arbitrary mathematical preference, but a consequence of requiring reconstruction stability under decomposition-invariant admissibility conditions. Quadratic geometry therefore emerges as the natural reconstruction geometry associated with admissibility-preserving projection structure.

The widespread appearance of L^2-type methods throughout physics may thus reflect a deeper geometric necessity tied to stable admissibility reconstruction rather than historical mathematical convention alone.

9.

Representation Rather Than Ontology

The distinction between reconstruction geometry and ontology is essential to the present framework. The argument developed here does not treat quadratic residual structure as evidence that reality dynamically minimizes errors, computes least-squares solutions, performs iterative optimization, or evolves through physical relaxation procedures. Such interpretations would improperly elevate representational reconstruction methods into ontological claims.

Instead, globally admissible relational structure is taken to exist all-at-once under closure constraints. Quadratic geometry appears only at the level of embedded representation, where local reconstruction attempts to approximate admissible structure through decomposition-independent projection. The resulting L^2-type geometry therefore belongs to the representational structure of admissibility reconstruction rather than to the underlying ontology itself.

This perspective preserves the adynamical character of the framework while maintaining representational neutrality across admissible reconstruction charts. Quadratic residual geometry emerges because embedded reconstruction requires stable decomposition, invariant residual partitioning, and well-conditioned projection structure, not because the universe itself is fundamentally performing optimization procedures.

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10. Representation-Independent Invariants

Representational neutrality removes any privileged reconstruction chart, but it does not eliminate invariant admissibility structure. Distinct reconstruction schemes may encode admissibility degeneration, curvature-like residual behavior, singular conditioning structure, or tangent-space collapse in different mathematical forms while still preserving common invariant scaling relations across admissible representations.

As a result, local divergence behavior may depend upon the reconstruction chart used to describe it, even when the underlying admissibility-conditioning structure remains representation-independent. This extends the broader representation-neutral reconstruction program developed within the RSE mathematical framework. [16] The same admissibility boundary may therefore appear differently across equivalent reconstruction schemes without altering the invariant geometric content associated with that boundary.

The situation is analogous to the distinction between coordinate singularities and curvature invariants in general relativity. No coordinate system possesses ontic privilege, yet invariant geometric structure remains well defined across admissible descriptions. Likewise, admissibility-conditioning structure may possess invariant content independent of the local reconstruction chart through which it is represented. [17]

This perspective preserves empirical and geometric content while avoiding commitment to chart realism. Reconstruction schemes function as representational encodings of admissible structure rather than as ontologically fundamental descriptions of reality itself.

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11. Relation to Hilbert Structure in Physics

The ubiquity of Hilbert-space methods in physics may reflect the natural geometry of admissibility reconstruction rather than an ontologically fundamental Hilbert structure. Under the framework developed here, the appearance of orthogonality, Fourier decomposition, spectral projection, quadratic action functionals, and least-squares reconstruction follows from the requirements of decomposition-independent admissibility geometry.

Once stable reconstruction requires invariant residual composition and orthogonal partitioning, Hilbert-like structure emerges as the natural representational language for embedded reconstruction of admissible relational configurations. The resulting mathematical framework supports stable projection geometry, spectral decomposition, and well-conditioned reconstruction across equivalent representations.

From this perspective, the recurring appearance of quadratic structures across physical theory acquires a unified geometric interpretation. The same reconstruction principles underlie a wide range of domains, including:

- quantum mechanics,
- field theory,
- statistical reconstruction,
- wave theory,
- and variational physics.

The common mathematical structure shared by these areas may therefore arise from a deeper requirement of admissibility-preserving reconstruction rather than from independent postulation within each individual theory.

The present argument supplies the representational background for quadratic structure; it does not by itself derive quantum conjugacy or the uncertainty relation, which follow in The Uncertainty Principle [18] from closure geometry, capacity structure, and complementary Lagrangian subspaces.

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12. Projection Residual Geometry (PRG)

Within Projection Residual Geometry (PRG), admissibility residual acquires a global metric interpretation through residual distance from the admissible manifold. The present work concerns the quadratic structure of that residual metric under decomposition-invariant reconstruction assumptions.

The local geometric refinements associated with admissibility restoration — including tangent-normal decomposition, curvature-like restoration structure, and conditioning geometry near admissibility boundaries — belong not to PRG itself but to Closure-Restoration Geometry (CRG). The present paper therefore addresses the residual metric shared by PRG and CRG rather than the local differential structure developed within CRG.

Within CRG, this supplies a candidate metric rationale for quadratic residual conditioning, orthogonal admissibility restoration, stable tangent-space decomposition, and locally conditioned projection geometry. The local structures themselves remain CRG results; the present paper explains why their shared residual metric is naturally L^2 under the assumptions stated in §3. The resulting framework allows admissibility deviation to be treated geometrically while preserving reconstruction covariance across equivalent representations. [2]

Within CRG, curvature-like reconstruction behavior and conditioning structure emerge from the local geometry of admissibility restoration near $\mathcal{M}$. PRG supplies the global residual metric structure shared by that local refinement.

As throughout the present work, these structures are understood representationally rather than ontically. PRG does not treat residual geometry, orthogonal restoration, or projection structure as fundamental constituents of reality, but as the natural geometric consequences of embedded admissibility reconstruction under closure and decomposition invariance.

12.1 Projection-Level Suppression and Effective Classicality

Projection-level reconstruction need not preserve all underlying structural distinctions. In weakly accessible reconstruction sectors, admissibility-preserving projection may compress substantial underlying variation into effectively reduced observational structure. This can produce residual attenuation, Gaussianization, or effective classicality despite the presence of richer underlying admissibility structure.

A useful example appears in recent analyses [13] of axion dark matter detection, where detectors couple not to the full underlying field configuration but to effective projected

reconstruction sectors constructed from weighted mode combinations. Under weak coupling and effective-mode averaging, intrinsically quantum structure becomes operationally inaccessible despite remaining formally present within the underlying description.

From the present perspective, such behavior may be understood geometrically as a conditioning property of admissibility-preserving reconstruction. Weakly conditioned projection sectors suppress distinguishability because accessible reconstruction charts no longer stably preserve the corresponding residual structure. Effective classicality therefore emerges not necessarily from ontological classicality, but from reconstruction geometry associated with admissibility-preserving projection under limited accessible support.

This interpretation aligns with Projection Residual Geometry [1], where observable reconstruction depends not only on underlying admissible structure, but also on the conditioning properties of the projection geometry relating embedded reconstruction sectors to the admissible manifold.

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13. Implications

The derivation developed here suggests that quadratic geometry may not require independent postulation within physical theory. Instead, Hilbert-like structure may emerge from the reconstruction requirements imposed by decomposition-independent admissibility conditions. Stable reconstruction geometry, orthogonal projection structure, and residual conditioning behavior all arise from the constraints governing embedded representations of globally admissible relational structure.

Under this interpretation, a substantial portion of the mathematical architecture appearing throughout physics acquires a unified geometric origin. Hilbert-like geometry is understood not as an ontologically fundamental substrate, but as the natural reconstruction language associated with stable admissibility-preserving representation. The recurring appearance of quadratic forms, orthogonal decomposition, spectral structure, and variational reconstruction methods may therefore reflect deep reconstruction constraints rather than separate formal postulates introduced independently across different theories. Related reconstruction-based derivations have been explored in closure-first treatments of uncertainty structure and invariant normalization geometry. [18, 14]

This perspective also reframes the interpretation of reconstruction singularities and degeneracy phenomena. Ill-conditioned reconstruction regions need not correspond to ontological discontinuities in the underlying admissible structure. Instead, they may indicate admissibility-conditioning boundaries, tangent-space degeneration, or the limits of stable local reconstruction near constrained admissibility-support regions.

More broadly, the framework suggests that many familiar geometric structures in physics may arise from the requirements of admissibility-preserving reconstruction under decomposition invariance. The geometry repeatedly encountered across physical theory may therefore reflect the representational structure of embedded reconstruction rather than the direct ontology of the underlying relational manifold itself.

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14. Conclusion

We have argued that global closure, decomposition independence, projection-level admissibility reconstruction, and stable conditioning requirements support quadratic residual geometry under the reconstruction assumptions specified in §3. Within this framework, L^2-type structure, orthogonal projection, inner-product geometry, and least-residual reconstruction emerge together as interconnected features of admissibility-preserving reconstruction, while local tangent-space refinement belongs to the CRG layer.

The resulting quadratic geometry is not introduced as an ontological primitive. Rather, it appears as the natural reconstruction geometry required for stable, decomposition-independent representation of globally admissible relational structure by embedded observers. Once admissibility invariance, reconstruction stability, direct-sum sector decomposition, and Pythagorean additivity are imposed, inner-product geometry follows by the Jordan–von Neumann route.

Representation neutrality removes any privileged reconstruction chart while preserving invariant admissibility structure across admissible representations. Reconstruction singularities, degeneracy behavior, and conditioning boundaries consequently admit a geometric interpretation tied to the stability limits of embedded reconstruction rather than to failures of the underlying ontology itself.

Under this interpretation, the widespread appearance of quadratic forms, orthogonal reconstruction procedures, spectral decomposition, and Hilbert-like geometry throughout physics reflects a common reconstruction principle grounded in admissibility invariance and closure structure. The mathematical structures repeatedly encountered across physical theory may therefore arise from the geometry of stable admissibility reconstruction rather than from independently postulated ontological primitives.

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