Structural Invariants and Dimensionless Ratios In a Closure-First Relational Framework

A principled account of invariants in admissible representations

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May 2026
DOI: 10.5281/zenodo.19868123
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Abstract

In a closure-first relational ontology, reality is identified with a single globally admissible structure satisfying consistency conditions. Any mathematical representation of such a structure introduces arbitrary descriptive elements—units, coordinates, and parameterizations—that are not ontic. This paper shows that admissible representations necessarily give rise to a class of dimensionless relational invariants, and that only such quantities can carry ontic significance.

We distinguish between representation-dependent invariants and structural invariants, defined as those that remain stable under admissible transformations of representation and under projection and restoration operations. Structural invariants therefore reflect properties of the underlying relational structure rather than artifacts of representation, and that the numerical realization of such invariants requires a global capacity normalization condition fixed by closure.

This framework provides a principled criterion for evaluating candidate constants and ratios, replacing model-dependent identification with a structural requirement of admissibility and representation-independence.

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1. Introduction

A recurring feature of theoretical physics is the appearance of dimensionless constants and ratios—quantities that appear independent of units and carry apparent physical significance. However, such quantities are often identified through exploratory or constructive methods: building a model, computing outputs, and searching for numerical agreement with known constants.

This raises a foundational question:

> Why should dimensionless invariants exist at all?

Rather than treating invariants as empirical discoveries within specific models, we seek a principled explanation for their necessity. In a closure-first relational framework, where reality is identified with a single globally admissible structure and representations are secondary, the existence of dimensionless relational invariants is not contingent—it follows from the structure of admissible representation.

In such a framework, representations introduce arbitrary choices of scale, units, and parameterization. Any quantity that depends on these choices is representation-dependent and cannot carry ontic significance. It follows that only quantities invariant under admissible transformations of representation—specifically, dimensionless relational ratios—can reflect the underlying structure.

The purpose of this paper is to:

- establish the necessity of dimensionless relational invariants from closure-first principles,
- define classes of such invariants within admissible representations,
- distinguish structural invariants from representation-dependent quantities,
- and provide a criterion for evaluating the physical significance of candidate ratios.

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2. Ontological Foundation

2.1 Closure-First Ontology

We adopt a closure-first ontology in which:

- Reality is a **single globally admissible structure**.
- Admissibility is defined by global consistency conditions (AGC, with RCCs as representational constraints). In boundary-operator representations, this closure requirement is expressed by BBP: boundary contributions must close without uncancelled second-boundary residue.
- Admissibility is **binary**: a configuration either satisfies these conditions or does not exist.

There is no notion of:

- partial admissibility,
- evolution toward admissibility,
- or approximation at the ontic level.

The configuration is not generated sequentially; it is selected all at once by global consistency.

2.2 Relational Structure

The ontology is fundamentally relational:

- No primitive objects or substances are assumed.
- Structure consists entirely of relations and their global coherence.
- Identity is associated with closure-class continuity within the admissible structure.

All apparent object-level descriptions arise as representational encodings of relational structure.

2.3 Consequence

Any representation of this structure necessarily introduces:

- coordinate systems,
- unit choices,
- normalization conventions.

These elements are not ontic; they are artifacts of representation.

Therefore:

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Any quantity with ontic significance must be invariant under admissible transformations of representation, including rescaling (NPRF) and reparameterization.

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3. Admissible Representations

3.1 Definition

An **admissible representation** is a mapping:

$$\mathcal{R}: \mathcal{S} \to \mathcal{C} \tag{3.1}$$

from the underlying relational structure $\mathcal{S}$ into a mathematical configuration space $\mathcal{C}$, such that:

- global consistency conditions (AGC) are preserved,
- relational structure is faithfully encoded,
- admissibility is maintained under the representation.

The configuration space $\mathcal{C}$ is an ambient space in which not all configurations are admissible; admissible representations map the underlying structure into the admissible subset of $\mathcal{C}$.

3.2 Properties

Admissible representations:

- may differ in coordinate systems, scaling, or parameterization (RN),
- are equivalent under admissible transformations such as global rescaling (NPRF),
- must preserve closure conditions and relational coherence,
- may admit non-unique realizations…which, in representations admitting an admissibility-relevant distance structure, are handled by projection-based methods (PRG [1]).

Thus, multiple admissible representations may exist for the same underlying relational structure, forming an equivalence class under admissible transformations.

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4. Representation vs Ontology

4.1 Projection and Residual

Representations operate in an ambient configuration space in which not all configurations satisfy admissibility. The admissible set $\mathcal{A}_{adm}$ is defined by global closure constraints.

Projection operators map configurations in the ambient space to the admissible set:

$$P_{\mathcal{A}_{adm}}: C \mapsto \hat{C} \tag{4.1}$$

We define the projection residual:

$$R(C) = \| C - P_{\mathcal{A}_{adm}}(C) \| \tag{4.2}$$

as a measure of deviation from admissibility. This residual is defined in the ambient configuration space; on the admissible set, it vanishes identically.

The residual is not an ontic quantity; it reflects only the mismatch between a representation and the admissible relational structure. A configuration with nonzero residual is not partially real—it is simply not admissible.

Quantities whose values depend on the residual are representation-dependent. Structural invariants are precisely those quantities for which the projection residual vanishes identically across admissible representations.

4.2 Projection Stability

Projection stability characterizes how sensitively the projection operator responds to perturbations in representation. It is governed by properties such as local conditioning of the projection map (e.g., Jacobian or Hessian structure in PRG [1]/CRG [2]).

Projection stability is not a property of the underlying relational structure, but of the representation used to encode it. Instabilities reflect poor conditioning of the representation, not ontic variability.

Stable invariants are those quantities whose values remain unchanged under perturbations that are removed by admissibility projection. In this sense, projection stability provides an operational criterion for distinguishing structural invariants from representation-dependent quantities.

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The local restoration version of this stability condition is developed in Closure-Restoration Geometry [2], where poor conditioning appears as path-dependent restoration, admissibility connection curvature, and holonomy of local correction maps.

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5. Necessity of Dimensionless Relational Invariants

5.1 Derivation from Closure-First Principles

We derive the necessity of dimensionless relational invariants directly from the foundational structure.

1. By RN and NPRF, representation-specific units and decompositions are not ontic; by NOE, no additional intrinsic scale is admitted unless required by closure.
2. By NPRF, admissible representations related by global rescaling are equivalent. Since admissible representations related by global rescaling are equivalent, any ontically meaningful quantity must be invariant under such transformations.
3. By LPS, only minimal structure required for admissibility is realized.
4. Therefore, any ontically meaningful quantity must be invariant under global rescaling of representation.

Under such rescaling, any quantity carrying dimension changes unless it appears as a ratio.

Therefore: Only dimensionless relational ratios are invariant under admissible rescalings, and thus only such quantities can carry ontic significance**.**

5.2 Interpretation

Dimensionless relational invariants are not special features discovered within particular models, but necessary consequences of representing a scale-free relational structure.

Any representation introduces arbitrary choices of units, normalization, and parameterization. Quantities that depend on these choices are representation-dependent and cannot carry ontic meaning.

Dimensionless relational ratios, by contrast, remain invariant under admissible transformations of representation. They reflect the underlying relational structure rather than the descriptive framework used to encode it.

Thus, invariants are not selected by empirical coincidence or model construction; they are the only quantities that can survive the representational freedom permitted by the framework.

5.3 Connection to Capacity Structure

In the mathematical model (MM) [3], relational structure is encoded by the capacity tensor Ξ, with normalized ratios:

$$\mathbb{J}_\alpha = \frac{\Xi_{\alpha\alpha}}{\mathbb{I}_0}. \tag{5.1}$$

The notation $\mathbb{J}_\alpha$ denotes normalized sector-capacity ratios. It should not be confused with the Jarlskog invariant J used in the 3-Mode Coherence Sector paper [4]. Where ambiguity may arise, these ratios may equivalently be denoted $\hat{\xi}_\alpha$. These ratios are invariant under global rescaling of Ξ, reflecting the NPRF equivalence class of admissible representations.

However, the normalization scale $\mathbb{I}_0$ is not introduced as a purely mathematical convenience. The capacity tensor admits a projective scaling freedom that leaves relational structure invariant but alters physical magnitudes. In a closure-first, representation-neutral framework, such freedom cannot remain unconstrained without introducing unphysical degrees of freedom.

Therefore, the admissible configuration must fix a global scalar invariant of the capacity structure. This invariant, denoted $\mathbb{I}_0$, represents the total coherence capacity invariant of the admissible configuration. In any given representation, it appears as a normalization of the capacity tensor, but its value is determined by global closure conditions rather than by choice.

This lifts the projective freedom inherent in the internal sector and provides the mechanism by which invariant structures acquire definite numerical values.

Thus, the appearance of dimensionless ratios in MM [3] is not an assumption, but a necessary consequence of the closure-first framework.

5.4 Capacity Normalization and Numerical Realization

The preceding derivation establishes that only dimensionless relational ratios can carry ontic significance within admissible representations. However, such ratios determine only relative structure; they do not fix absolute magnitudes.

The capacity tensor therefore admits a projective freedom under global rescaling:

$$\Xi \to c\,\Xi, \tag{5.2}$$

which leaves all dimensionless invariants unchanged.

In a closure-constrained framework, this freedom cannot remain physically unconstrained. A global rescaling that affects observable magnitudes would introduce a representational degree of freedom inconsistent with RN/NPRF and LPS/NOE.

It follows that the admissible configuration must fix a global capacity invariant:

$$\mathbb{I}_0 = \text{Invariant}\ (\Xi), \tag{5.3}$$

which determines the normalization scale of the capacity tensor. The existence of $\mathbb{I}_0$ is required to eliminate projective freedom; its explicit determination is not addressed here.

In a given representation, this invariant may be expressed through a scalar constraint:

$$\text{Tr}\ (\Xi) = \mathbb{I}_0, \tag{5.4}$$

or an equivalent normalization condition. Such expressions are representation-dependent realizations of the underlying invariant, not its definition.

The role of $\mathbb{I}_0$ is therefore twofold:

- it removes the projective scaling freedom of admissible representations,
- and it provides the mechanism by which relational invariants acquire definite numerical values.

This establishes a separation between:

- structural determination of invariant form (Sections 5.1–5.3), and
- global determination of invariant magnitude through capacity normalization.

The explicit determination of $\mathbb{I}_0$ from closure conditions remains an open problem within the framework.

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6. Classes of Invariants

6.1 Necessity of Classes

Admissibility constraints act on relational structure rather than on individual numerical values. Any admissible construction generates quantities that are invariant under representational transformations (e.g., rescaling, reparameterization). Because these transformations form equivalence classes, invariant quantities necessarily arise not as isolated values but as families of equivalent expressions.

Therefore, invariants do not appear singly; they appear as classes defined by a common relational construction rule.

6.2 Definition

A class of dimensionless relational invariants is a set of quantities generated by a common relational construction rule and grouped by equivalence under admissible transformations of representation.

Two quantities belong to the same class if they:

- arise from the same relational construction, and
- are related by admissible transformations (e.g., scaling, coordinate change, normalization).

Formally, a class of invariants may be defined as an equivalence class under admissible transformations: $I \sim I'$ if I' is obtained from I by an admissible transformation of representation.

6.3 Construction Rules

Typical invariant classes arise from:

- ratios of relational magnitudes,
- normalized differences,
- projection-based overlap measures,
- comparisons between local and global structure.

Each construction rule generates a family of equivalent invariants across admissible representations.

6.4 Example

Let $f(x)$ be a relational quantity and λ a representation-dependent scaling factor. Then:

$$I(x) = f(x)/\lambda \tag{6.1}$$

defines a class of invariants under admissible rescaling transformations.

A specific numerical value (e.g., 1.9171) is not itself fundamental; it is one realization within the class. Its significance depends on whether it remains stable across admissible representations.

6.5 Consequence

Because admissible representations form equivalence classes under transformation, invariant classes are unavoidable. The task of physical interpretation is therefore not to discover isolated numerical constants, but to identify which invariant classes correspond to stable relational structure.

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7. Structural vs Representation-Dependent Invariants

7.1 Hierarchy

We distinguish four levels of quantities arising in admissible representations:

Level	Description
Raw quantities	representation-dependent
Dimensionless invariants	unit-independent
Classes of invariants	structurally defined families
Structural invariants	representation-independent

This hierarchy reflects increasing independence from representational choices, culminating in quantities that depend only on the underlying relational structure.

7.2 Structural Invariant Definition

A structural invariant is a dimensionless relational quantity that remains unchanged across all admissible representations of the same underlying structure and is stable under admissibility projection and restoration.

Equivalently, a structural invariant is invariant under all admissible transformations of representation, including rescaling (NPRF), reparameterization (RN), and admissibility-preserving operations (PRG/CRG).

Thus, structural invariants are not merely unit-independent; they are quantities that persist under all admissible transformations of representation and under the operations required to enforce admissibility.

7.3 Criterion for Physical Significance

A candidate invariant is physically meaningful only if it satisfies all of the following:

- it is dimensionless,
- it is relational (constructed from relations within the structure),
- it is admissibility-compatible (defined on the admissible set),
- it is stable under admissible transformations of representation,
- and it remains invariant under projection and restoration operations.

Quantities that fail any of these conditions are representation-dependent and do not reflect the underlying relational structure. Structural invariants therefore define the only quantities that can be consistently interpreted as physical within a closure-first relational framework.

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8. Grassmannian Formulation

8.1 Relational Structure as Subspaces

Relational structure can be represented as subspaces in a Grassmannian manifold:

$$V_i \in Gr(k, n) \tag{8.1}$$

Each subspace V_i corresponds to an admissible relational configuration or component of the global structure. The Grassmannian thus provides a natural configuration space for representing relational structure without introducing coordinate-dependent primitives. Only those subspaces satisfying the global closure constraints belong to the admissible subset of the Grassmannian representation.

8.2 Indexing and Adjacency

Indexing within this space is defined by geometric relations between subspaces, including:

- projection overlap,
- principal angles,
- subspace intersection.

These relations define adjacency and ordering without reference to an external temporal parameter. Apparent ordering arises from relational proximity in the Grassmannian, not from ontic evolution.

8.3 Invariants in Grassmannian Terms

Within this representation, invariants arise from geometric relations between subspaces. Examples include principal angle spectra, projection norms, andoverlap ratios.

These quantities are inherently dimensionless and relational. They are invariant under admissible transformations of representation and correspond to projection-stable quantities as defined in Sections 4–7.

8.4 Key Insight

Invariants arise from geometric relations between subspaces, not from temporal evolution or observer-dependent slicing. The Grassmannian formulation makes explicit that structural invariants are properties of relational geometry within the admissible set.

9. Observables and Projection

9.1 Observations as Restricted Access

Observers do not access the full relational structure $\mathcal{X}$, but only projection-limited representations of the underlying relational structure. Observation corresponds to projection onto a limited family of subspaces within the Grassmannian representation.

Thus, what is observed is not the full structure, but a representation constrained by the observer's accessible relational domain.

9.2 What Survives Projection

Different projections produce different representations of the same underlying structure. However, quantities that depend on the choice of projection are representation-dependent.

Structural invariants are precisely those quantities that remain unchanged under all admissible projections. Equivalently, they are quantities whose values are preserved under projection and restoration operations (PRG/CRG).

9.3 Clarification

Invariants are not the difference between reality and experience. They are not residual artifacts of observation.

Rather, invariants are the quantities that remain unchanged across all admissible representations and projections of the same underlying relational structure.

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10. Implications

10.1 Numerical Scanning

Searching for constants by scanning model outputs lacks a principled basis. Such approaches may yield empirically useful approximations, but they do not provide a principled justification for the existence or uniqueness of the resulting constants. Without a prior derivation of invariant structure, any numerical agreement is ambiguous, since representation-dependent quantities can produce arbitrary matches.

10.2 Constructive Selection

Selecting structures or constraints because they reproduce known results does not constitute derivation. A principled framework must demonstrate why such structures are required by admissibility, not merely that they are compatible with observations.

10.3 Proper Program

A principled approach proceeds by:

1. establishing the necessity of invariant classes from the underlying ontology,
2. identifying candidate invariants within admissible representations,
3. testing their stability under admissible transformations, projection, and restoration,
4. calibrate against empirical data at the representational level,
5. test stability under admissible transformations.

Only invariants that survive all such tests can be considered structurally meaningful. This restriction does not prohibit phenomenological calibration at the representational layer. Empirical values may be used to identify or test a representation once the relevant invariant structure has been derived. What is excluded is treating empirically selected numerical constants as ontic primitives.

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11. Example Application

Consider a dimensionless ratio derived from a specific representation (e.g., a lattice discretization) [4]. Such ratios are representation-dependent unless shown to persist across admissible transformations and discretization choices. Its appearance alone does not establish physical significance.

To qualify as a structural invariant, it must:

- remain stable under changes in discretization,
- remain invariant under normalization and rescaling,
- persist across admissible representations of the same underlying structure,
- and remain unchanged under projection and restoration operations.

Only quantities satisfying these conditions can be interpreted as reflecting the underlying relational structure.

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12. Conclusion

We have shown that:

- dimensionless relational invariants are necessary in any admissible representation of a closure-first relational ontology,
- such invariants arise as equivalence classes defined by relational construction rules,
- only those invariants that remain stable under admissible transformations, projection, and restoration qualify as structural invariants.

The key distinction is:

> The existence of invariants follows from the structure of admissible representation of a closure-first relational ontology; the identification of any specific invariant as fundamental requires demonstration of representation-independence and projection stability.

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